20550
Math Calculus Name:______________________________

Final Exam *December 16, 2005* Section:______________________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 20 multiple choice questions. Please sign the honor statement if you agree:

"I strictly followed the Notre Dame Honor Code during this test."

Your Signature ________________________

1.	a	b	c	d	e	11.	a	b	c	d	e
2.	a	b	c	d	e	12.	a	b	c	d	e
3.	a	b	c	d	e	13.	a	b	c	d	e
4.	a	b	c	d	e	14.	a	b	c	d	e
5.	a	b	c	d	e	15.	a	b	c	d	e
6.	a	b	c	d	e	16.	a	b	c	d	e
7.	a	b	c	d	e	17.	a	b	c	d	e
8.	a	b	c	d	e	18.	a	b	c	d	e
9.	a	b	c	d	e	19.	a	b	c	d	e
10.	a	b	c	d	e	20.	a	b	c	d	e

1. Find the maximum rate of change of $f(x, y) = x^2y + 2y$ at the point $(-1, 2)$ and the direction in which it occurs.

(a) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle -4, 3\rangle$.

(b) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle 4, -3\rangle$.

(c) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle 4, 3\rangle$.

(d) The maximum rate of change is 6 in the direction $\frac{1}{5}\langle -4, 3\rangle$.

(e) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle -3, 4\rangle$.

2. Let S be the part of cylinder $y^2 + z^2 = 1$, with $z \geq 0$, and $0 \leq x \leq 1$, and let S have the upward orientation. Determine which of the following equals $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle 0, 0, z \rangle$.

(a) $\int_0^1 \int_{-1}^1 \sqrt{1 - y^2} dydx.$

(b) $\int_0^1 \int_{-1}^1 (1 - y^2) dydx$

(c) $\int_0^1 \int_{-1}^1 (1 - x^2) dydx$

(d) $\int_0^1 \int_{-1}^1 [\sqrt{1 - y^2}\ ]^{-1} dydx$

(e) $\int_0^1 \int_{-1}^1 \sqrt{1 - x^2} dydx$

3. Let C be the triangle with vertices $(0,0)$, $(1,1)$, and $(2,0)$ oriented counterclockwise. Compute

$$\int_C [\cos x^{100} + x^4 y^5]dx + [\sin(e^y) + x^5 y^4]dy$$

(a) 0 (b) $\frac{5}{4}$ (c) -1 (d) $\frac{4}{5}$ (e) 1

4. Let C be the curve $\mathbf{r}(t) = \langle t, \cos(2t), 1 + \sin(3t)\rangle$, $0 \leq t \leq \frac{\pi}{2}$, and let

$$\mathbf{F}(x, y, z) = \langle y(2x + z), x(x + z) - z, y(x - 1) + 2z\rangle.$$

Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Hint: Find f with $\mathbf{F} = \nabla f$).

(a) $-\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2} - 1$ (c) $\frac{\pi}{2}$ (d) 0 (e) $1 - \frac{\pi}{4}$

5. Find the absolute minimum of $f(x, y) = x^2 + 2y^2 + 4y - 2$ on the disk $x^2 + y^2 \leq 4$.

(a) -4 (b) -6 (c) 0 (d) -2 (e) 14

6. Find the scalar projection, $comp_{\mathbf{v}}(\mathbf{w})$, of the vector $\mathbf{w} = \langle 1, 1, 2 \rangle$ onto the vector $\mathbf{v} = \langle 2, -2, 1 \rangle$.

(a) 2 (b) $\frac{2}{3}$ (c) $\frac{2}{\sqrt{6}}$ (d) 1 (e) $-\frac{2}{3}$

7. Determine two vectors that are tangent to the surface $\mathbf{r}(u, v) = \langle vu^2 - 2u, uv^2 - v, uv\rangle$ at the point $(0, 1, 2)$.

(a) $\langle 0, 1, 1\rangle, \langle 4, 1, 1\rangle$ (b) $\langle 2, 4, 2\rangle, \langle 1, 3, 1\rangle$ (c) $\langle 2, 1, 1\rangle, \langle 4, 3, 2\rangle$

(d) $\langle 0, 2, -1\rangle, \langle 1, -6, 3\rangle$ (e) $\langle 1, 2, -1\rangle, \langle 1, -2, 1\rangle$

8. Evaluate $\int\int\int_E 3e^{[(x^2+y^2+z^2)^{\frac{3}{2}}]}dV$ where E is the upper *half* of the ball radius 1 centered at the origin.

(a) π (b) $4\pi(e-1)$ (c) 2π (d) $2\pi(e-1)$ (e) $\pi(e-1)$

9. If $z = f(x, y)$, $x = u^2 + v^2$ and $y = u^2 - v^2$, find $\dfrac{\partial^2 z}{\partial u \partial v}$.

(a) $4uv(f_{xx} - f_{yy})$ (b) $4uv(f_{xx} + 2f_{yy} + f_{yy})$ (c) $2(f_{xx} + f_{yy})$

(d) $4uv(f_{xx} + f_{yy})$ (e) $4uv(f_{xx} + 2f_{yy} - f_{yy})$

10. Find a direction vector for the line of intersection of the planes $x + y + 2z = 1$ and $3x - y = 0$.

(a) $\langle 1, 3, -2\rangle$ (b) $\langle 3, 1, -2\rangle$ (c) $\langle 3, -2, 1\rangle$ (d) $\langle -2, 3, 1\rangle$ (e) $\langle 1, 3, 2\rangle$

11. Calculate the arc length of the helix parameterized by $\mathbf{r} = \langle -3t, 4\cos t, -4\sin t \rangle$ for $0 \leq t \leq \pi$

(a) 12π (b) 2π (c) 5π (d) 0 (e) 10π

12. Find $\displaystyle\int\!\!\int_S \mathbf{F} \cdot d\mathbf{S}$ where $F(x, y, z) = \langle xy, \frac{3}{4}y, -zy \rangle$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ with the outward orientation.

(a) 2π (b) 16π (c) π (d) 0 (e) 8π

13. Express the area between $x^2 + \frac{y^2}{9} = 1$ and $x^2 + \frac{y^2}{9} = 9$ as an integral, using the substitution $x = r\cos\theta$ and $y = 3r\sin\theta$.

(a) $\int_0^{2\pi}\int_0^3 3r\,dr\,d\theta$ (b) $\int_0^{2\pi}\int_1^3 9r\,dr\,d\theta$ (c) $\int_0^{2\pi}\int_1^3 3r\,dr\,d\theta$

(d) $\int_0^{2\pi}\int_1^3 3r^2\,dr\,d\theta$ (e) $\int_0^{2\pi}\int_1^9 3r\,dr\,d\theta$

14. Let C be the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 2y + 3$ oriented counter-clockwise with the normal upwards. Use Stokes Theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = \langle 2e^y - z, \cos(yz), xe^y \rangle.$$

(a) 0 (b) $\frac{\pi}{\sqrt{5}}$ (c) 2π (d) $\sqrt{5}\pi$ (e) $\frac{2\pi}{\sqrt{5}}$

15. Determine which of the following integrals gives the area of the region in the xy-plane below the x-axis above $y = x^2 - 2$ and to the left of $y = -2x - 2$.

(a) $\displaystyle\int_{-\sqrt{2}}^{0} \int_{-\sqrt{y+2}}^{-\frac{y}{2}-1} dx dy$ (b) $\displaystyle\int_{-2}^{0} \int_{\sqrt{y+2}}^{-\frac{y}{2}-1} dx dy$ (c) $\displaystyle\int_{-2}^{0} \int_{-2x-2}^{x^2-2} dy dx$

(d) $\displaystyle\int_{-2}^{0} \int_{-\sqrt{y+2}}^{1-\frac{y}{2}} dx dy$ (e) $\displaystyle\int_{-2}^{0} \int_{-\sqrt{y+2}}^{-\frac{y}{2}-1} dx dy$

16. Evaluate $\displaystyle\int_C (1 + x^2 y) ds$ where C is the upper half of the unit circle $x^2 + y^2 = 1$.

(a) 1. (b) $\dfrac{2}{3}$. (c) 0. (d) 2π. (e) $\pi + \dfrac{2}{3}$.

17. Determine which of the following integrals gives the volume of the region bounded by the cylinder $x^2 + y^2 = 1$, and the planes $z = 0$ and $x + z = 1$.

(a) $\int_0^{2\pi} \int_0^1 \int_0^{1-r\cos\theta} r\,dz\,dr\,d\theta$

(b) $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{1-x} dz\,dx\,dy$

(c) $\int_0^{2\pi} \int_0^1 \int_0^{1-r\cos\theta} dz\,dr\,d\theta$

(d) $\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{1-r\cos\theta} r\,dz\,dr\,d\theta$

(e) $\int_0^{\pi} \int_0^1 \int_0^{1-r\cos\theta} r\,dz\,dr\,d\theta$

18. Find surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.

(a) $\int_0^{2\pi} \int_0^2 r\sqrt{1+4r^2}drd\theta$

(b) $\int_0^{2\pi} \int_0^4 r\sqrt{1+4r^2}drd\theta$

(c) $\int_0^{2\pi} \int_0^1 r\sqrt{1+4r^2}drd\theta$

(d) $\int_0^{\pi} \int_0^2 r\sqrt{1+4r^2}drd\theta$

(e) $\int_0^{2\pi} \int_0^2 \sqrt{1+4r^2}drd\theta$

19. A particle starts from rest at time $t = 0$ at the origin $(0, 0, 0)$. It then begins to move with acceleration $\mathbf{a}(t) = \langle 1, 6t, 12t^2 \rangle$. Find the time, if ever, at which the particle passes through the point $(1, 1, 1)$.

(a) never (b) $t = 2$ (c) $t = 6$ (d) $t = 3$ (e) $t = 1$

20. Find the minimum of the function $f(x, y, z) = x^2 + y^2 + 2z^2$ on the surface $x^2y^2z = 16$.

(a) 14 (b) 12 (c) 8 (d) 16 (e) 10

Answer Key 1

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5.	•	b	c	d	e	15.	a	b	c	d	•
6.	a	•	c	d	e	16.	a	b	c	d	•
7.	a	b	•	d	e	17.	•	b	c	d	e
8.	a	b	c	•	e	18.	•	b	c	d	e
9.	•	b	c	d	e	19.	•	b	c	d	e
10.	•	b	c	d	e	20.	a	b	c	d	•