

Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

Math 10250 - Elements of Calculus I

Final Exam - Fall 2005

Friday, December 16, 1:45-3:45 PM

---

This Examination contains **30** multiple choice problems with no partial credit. Each is worth 5 points for a total of 150 points. Books and notes are not allowed. You may use your calculator. Also, **show your Work!**

Record your answers to these problems by placing an  $\times$  through one letter for each problem below:

- |  |  |  |
|--|--|--|
| 1. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 11. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 21. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 2. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 12. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 22. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 3. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 13. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 23. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 4. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 14. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 24. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 5. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 15. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 25. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 6. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 16. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 26. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 7. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 17. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 27. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 8. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 18. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 28. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 9. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e  | 19. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 29. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |
| 10. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 20. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e | 30. <input type="checkbox"/> a <input type="checkbox"/> b <input type="checkbox"/> c <input type="checkbox"/> d <input type="checkbox"/> e |

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

\_\_\_\_\_

GOOD LUCK

### 30 Multiple Choice Questions (5 Points Each)

1. Find the equation of the line passing through  $(-1, 2)$  and  $(3, -2)$ .

(a)  $y = -x + 1$

(b)  $y = -2x + 2$

(c)  $y = -x - 1$

(d)  $y = -\frac{1}{2}x + 1$

(e)  $y = -2x - 1$

2. Find the inverse of the function  $f(x) = 2^{x-1}$

(a)  $g(x) = \ln x + \ln 2 + 1$

(b)  $g(x) = \ln\left(\frac{x}{2}\right) + 1$

(c)  $g(x) = (\log_2 x) + 1$

(d)  $g(x) = 2 \ln(x - 1)$

(e)  $g(x) = \log_2(x + 1)$

3. Find the limit

$$\lim_{h \rightarrow 0} \frac{h}{1 - \sqrt{1+h}}$$

(a) 2

(b) 0

(c) 1

(d) -2

(e) -1

4. Suppose that you put \$1000 in an account paying 4% annual interest compounded monthly. Find the balance at the end of 3 years.

(a)  $1000 \left(1 + \frac{0.4}{12}\right)^{24}$

(b)  $1000 \left(1 + \frac{0.04}{12}\right)^{36}$

(c)  $1000 \left(1 + \frac{4}{12}\right)^{36}$

(d)  $1000 \left(1 + \frac{0.04}{12}\right)^{12}$

(e)  $1000 \left(1 + \frac{0.04}{12}\right)^3$

5. Find the limit:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n$$

(a)  $e$

(b) 1

(c)  $\infty$

(d) 0

(e)  $e^2$

6. The initial amount of a certain radioactive substance is 8 grams. After 16 days there are only 0.5 grams. What is the half-life of the substance? (All answers are in days.)

- (a) 4                      (b) 2                      (c) 0.5                      (d) 1                      (e) 8

7. Which of the following is the instantaneous rate of change of  $y = e^{3x^2}$  at  $x = 1$ ?

- (a)  $6e^3$                       (b)  $e^3$                       (c)  $3e^3$                       (d)  $3e$                       (e) 1

8. Which of the following limit represents  $f'(2)$ , where  $f(x) = x^5$ ?

- (a)  $\lim_{h \rightarrow 0} \frac{32 - h^5}{h}$                       (b)  $\lim_{h \rightarrow 0} \frac{(2 + h)^5 - 32}{h}$                       (c)  $\lim_{h \rightarrow 0} \frac{(x - 2)^5 - 32}{h}$
- (d)  $\lim_{h \rightarrow 0} \frac{(h - 2)^5 - 32}{h}$                       (e)  $\lim_{h \rightarrow 0} \frac{32 - (2 + h)^5}{h}$

9. Find the equation of the tangent line to  $y = x^4 - x$  at  $x = 2$ .

- (a)  $y - 31 = 14(x - 2)$                       (b)  $y - 2 = 31(x - 14)$                       (c)  $y - 14 = 2(x - 31)$
- (d)  $y - 14 = 31(x - 2)$                       (e)  $y - 31 = 2(x - 14)$

10. Find all points where the tangent line to graph of  $y = x \ln x - 2x$  is horizontal.

- (a)  $x = 0$                       (b)  $x = 1$                       (c)  $x = e$                       (d)  $x = 2$                       (e)  $x = \frac{e}{2}$

11. Let  $f(x) = \frac{x^2 + c}{x + 1}$  where  $c$  is a constant. What is  $f'(x)$ ?

(a)  $\frac{-x^2 - 2x + c}{(x + 1)^2}$

(b)  $\frac{3x^2 + 2x + c}{(x + 1)^2}$

(c)  $\frac{x^2 + 2x - c}{(x + 1)^2}$

(d)  $\frac{-3x^2 - 2x - c}{(x + 1)^2}$

(e) 0

12. Find  $\frac{dy}{dx}$  if  $x^2y + xy^4 = 2$ .

(a)  $\frac{2xy + y^4}{x^2 - 4xy^3}$

(b)  $\frac{2xy - y^4}{x^2 + 4xy^3}$

(c)  $\frac{2xy - y^4}{x^2 - 4xy^3}$

(d)  $\frac{2xy - y^4}{-x^2 - 4xy^3}$

(e)  $\frac{-2xy - y^4}{x^2 + 4xy^3}$

13. Let  $f(x)$  be a function such that  $f(3) = 4$  and  $f'(3) = 2$ . Use linear approximation to estimate  $f(3.2)$ .

(a) 4.4

(b) 4.2

(c) 4

(d) 4.6

(e) 4.1

14. Let  $f(x)$  and  $g(x)$  be functions whose graphs are

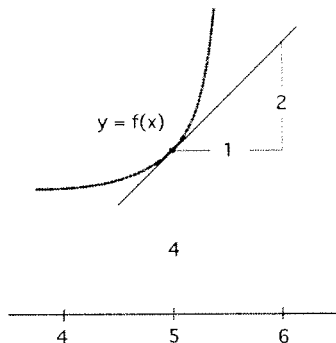


Figure 1

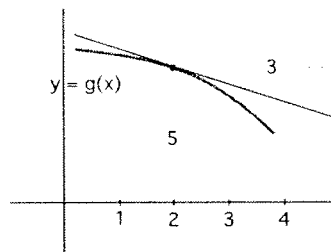


Figure 2

Compute the derivative of  $f(g(x))$  at  $x = 2$ .

- (a)  $-\frac{2}{3}$                       (b)  $\frac{5}{3}$                       (c)  $-\frac{5}{3}$                       (d) 20                      (e)  $\frac{2}{5}$

15. A pebble is dropped in the middle of a pond, creating an expanding circular ripple. Suppose that at a certain moment the radius is 10 ft. and is increasing at a rate of 2 ft/sec. How quickly is the area increasing at the moment? Remember that  $A = \pi r^2$ , where  $A$  is the area and  $r$  is the radius.

- (a)  $20\pi$  ft<sup>2</sup>/sec                      (b)  $40\pi$  ft<sup>2</sup>/sec                      (c)  $10\pi$  ft<sup>2</sup>/sec                      (d) 4 ft<sup>2</sup>/sec                      (e) 2 ft<sup>2</sup>/sec

16. Where does the global maximum of the function  $f(x) = \frac{\ln x}{x}$  on the interval  $(0, \infty)$  occur?

- (a)  $x = e^2$                                       (b)  $x = 1$                                       (c)  $x = e$   
 (d)  $x = \sqrt{e}$                                       (e)  $f(x)$  has no global maximum on this interval

17. Find all the critical points of the function  $f(x) = xe^x$ .

(a)  $x = 0$

(b)  $x = 1$

(c)  $x = -1$

(d)  $x = -1, 0$

(e) There are no critical points.

18. Suppose that the graph of the **derivative** of the function  $f(x)$  is given below. Which of the following statements is **NOT** true?

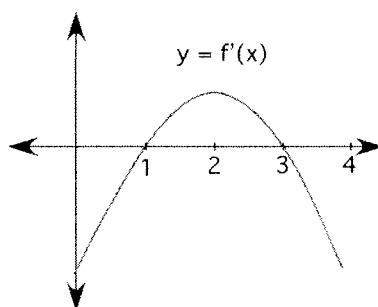


Figure 3

(a)  $f(x)$  has an inflection point at  $x = 2$

(b)  $f(x)$  has a local maximum at  $x = 3$

(c)  $f(x)$  is increasing on  $(1, 3)$

(d)  $f(x)$  is decreasing on  $(3, 4)$

(e)  $f(x)$  has a local maximum at  $x = 1$



22. Here is a partial table of the values of  $f'(x)$  and  $f''(x)$  for a certain function  $f(x)$ .

$x$	0	1	2	3	4	5	6
$f'(x)$	-1	0	1	0	-1	0	-1
$f''(x)$	0	1	0	0	0	-1	-2

Which of the following is NOT possible?

- (a)  $f$  has an inflection point at  $x = 4$
- (b)  $f$  has a local maximum at  $x = 1$
- (c)  $f$  has an inflection point at  $x = 3$
- (d)  $f$  has a local maximum at  $x = 3$
- (e)  $f$  has a local maximum at  $x = 5$

23. Suppose you estimate the area under the graph of  $y = 1/x$  for  $1 \leq x \leq 2$  by a Riemann sum using  $n = 4$  and left-hand endpoints. What would be the estimate?

- (a)  $\frac{319}{105}$
- (b)  $\frac{533}{840}$
- (c)  $\frac{106}{105}$
- (d)  $\frac{319}{420}$
- (e)  $\frac{107}{210}$

24. Find  $\int \frac{1}{t^2 - 9} dt$ .

- (a)  $\frac{1}{6} \ln |t - 3| - \frac{1}{6} \ln |t + 3| + c$
- (b)  $\frac{1}{3t} \ln |t^2 - 9| + c$
- (c)  $\frac{-3t}{(t^2 - 9)^2}$
- (d)  $\ln |t^2 - 9| + c$
- (e)  $\frac{1}{3} \ln |t - 3| + \frac{1}{3} \ln |t + 2| + c$



25. Find the area of the region enclosed by the curves  $y = 4 - x^2$  and  $y = x^2 - 4x + 4$  between their points of intersections.

- (a)  $\frac{1}{3}$                       (b)  $\frac{4}{3}$                       (c)  $\frac{8}{3}$                       (d) 12                      (e)  $-\frac{8}{3}$

26. Find the area under the graph of  $y = x^{1/3}$  for  $0 \leq x \leq 1$ .

- (a)  $\frac{2}{3}$                       (b)  $\frac{4}{3}$                       (c)  $\frac{1}{3}$                       (d) 1                      (e)  $\frac{3}{4}$

27. Figure 6 shows the graph of  $y = f(x)$ ,  $-2 \leq x \leq 6$ . Given that

$$\int_{-2}^1 f(x)dx = -2.8, \quad \int_1^3 f(x)dx = 1.2, \quad \int_1^6 f(x)dx = -3.5,$$

find the area of the region enclosed by the graph of  $f(x)$ ,  $-2 \leq x \leq 6$  and the  $x$ -axis.

**NOTE: Pay attention to the limits of integration**

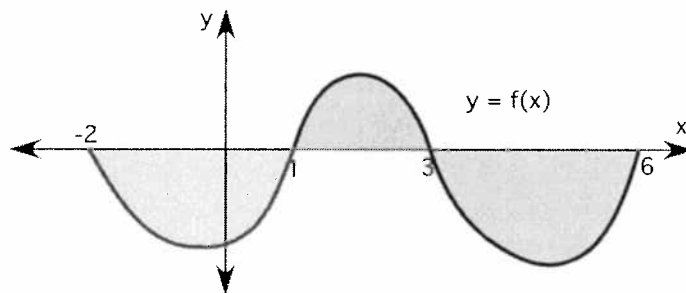


Figure 6

- (a) 7.5                      (b) 8.7                      (c) 5.1                      (d) 6.3                      (e) -5.1

28. Find  $\int x^2 \ln x \, dx$

(a)  $\frac{x^3}{3} (\ln x - \frac{1}{3}) + c$

(b)  $\frac{x^3}{3} (\ln x - 1) + c$

(c)  $\frac{x^2}{3} (\ln x - \frac{x}{3}) + c$

(d)  $x - \frac{x^2}{2} + c$

(e)  $2x(\ln x - 1) + c$

29. If  $g(t) = t^2 + \int_e^t (\ln x) \sqrt{1+x^3} dx$ , then  $g'(1)$  is:

(a)  $2 - \sqrt{e^3 + 1}$

(b) 2

(c)  $\frac{1}{3}$

(d)  $2e + \sqrt{e^3 + 1}$

(e)  $2 - e - \sqrt{e^3 + 1}$

30. A company's marginal cost for producing  $x$  units per day of its products is given by the function  $MC(x) = 3x^2 - 10x + 80$  dollars per unit. If the company operates at the production level of 20 units per day, find the increase in cost involved in raising the level to 30 units per day.

(a)  $\frac{27,500}{3}$

(b) 32,500

(c) 17,300

(d)  $\frac{1,390}{3}$

(e)  $\frac{13,900}{3}$

## Math 10250 Sample Exam Solutions

---

1.  a  b  c  d  e
2.  a  b  c  d  e
3.  a  b  c  d  e
4.  a  b  c  d  e
5.  a  b  c  d  e
6.  a  b  c  d  e
7.  a  b  c  d  e
8.  a  b  c  d  e
9.  a  b  c  d  e
10.  a  b  c  d  e
11.  a  b  c  d  e
12.  a  b  c  d  e
13.  a  b  c  d  e
14.  a  b  c  d  e
15.  a  b  c  d  e
16.  a  b  c  d  e
17.  a  b  c  d  e
18.  a  b  c  d  e
19.  a  b  c  d  e
20.  a  b  c  d  e
21.  a  b  c  d  e
22.  a  b  c  d  e
23.  a  b  c  d  e
24.  a  b  c  d  e
25.  a  b  c  d  e
26.  a  b  c  d  e
27.  a  b  c  d  e
28.  a  b  c  d  e
29.  a  b  c  d  e
30.  a  b  c  d  e